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IMPORTANCE OF INDIRECT MEASUREMENT IN ELEMENTARY MATHEMATICAL TEACHING.

THE intensely practical spirit, characteristic of our time and land, has not been an unmixed blessing to American education. When a principle has been found to work well, to economize effort, in one field of activity, it is the genius of our people to carry it over into other fields. Educational practice has profited greatly during recent times by the readiness and the cleverness with which the American teacher perceives a successful principle in an adjoining field, adapts it to his own profession, and adopts it into his practice. The laboratory method of imparting truth did not wait long after science teachers had proved its value until alert teachers in numerous lines were availing themselves of its vitalizing influence. For years the most successful mathematical teachers have been pleading to deaf ears from considerations of both theory and practice for the general adoption of this method in the mathematical classroom. Only those mathematical teachers who depend for their professional standing upon the reputation of their institution can afford much longer to wrap themselves in the bandages of an unreasoning conservatism which closes all avenues of mental approach and destroys every possibility of professional improvement.

But the dictum that the end justifies the means, when transferred from the commercial to the educational field, becomes mischievous to a high degree. It is scarcely necessary to say that where the chief end of activity is morality this dictum is most pernicious. If, therefore, this maxim of the mart has any relevancy to mathematical teaching, this is the only line of teaching in which it would be tolerated for an hour.

The rapidly waning popularity which greets the deviceful ingenuity of the modern mathematical pedagogue disproves the pertinency of this dictum to mathematical teaching. Many are coming to realize that pedagogical air lines constructed to bear the learner most quickly and pleasurably to the delectable hills of mathematical learning are all of them narrow-gauge routes, over bumpy roadbeds, through stretches of monotonous number scenery alternating with districts permitting only the most contracted view of the vaster fields of mathematical truth.

The reason from the side of subject-matter for the inadequacy of the short-cuts to mathematics lies in the fact that they all build on fragmentary notions of the cultural and practical value of mathematics. From the side of method they fail signally in that they do not take the life and purpose of the pupil sufficiently into account. Most of the psychological methods of developing the subject in the pupil's mind select some interesting psychological aspect of numbers and undertake to magnify this aspect into the most distorted and oftentimes grotesque proportions.

The production of these fragmentary solutions of the problem of mathematical teaching a few years since had become an epidemic, not to say a contagion, among us. It was thought that this busy, buzzing age demanded short, sharp, and snap solutions of all its problems. A student of that time, belonging to a sophomore calculus class of young engineers, had become infected with the yearning for the "nutshell" class of solutions. The student in question was having difficulty with his subject. It is from this sort that the "nutshell" type is born. "Professor, what is the core principle of this 'stuff' at any rate?" he asked. "Can't you give it to me in a 'nutshell'?" On being told that the "core principle" of calculus, though marvelously fruitful once its full significance was grasped, was so general as to be of almost no practical value to the learner, he refused to be satisfied. "What is it?" he insisted. He was then told that it was "the idea of rates." It is needless to say he wanted an explanation of the explanation; whereupon he was told that this is what we are trying to give in the recitation room from day to day. Six months more hard work brought from him the confession that this *is* the "core principle," and, moreover, that this principle is too general to be reduced directly to terms of practice by the beginner.

Several years of close contact with numerous mathematical teachers of almost every department of school work lead the writer to believe that the cast of mind indicated by this student's questions is typical of the poorly equipped and more or less deviceful pedagogue (not teacher). The danger of this attitude of mind is that it does not grasp deeply enough to seize the veritable heart of the subject, and that a partial rather than the whole truth is usually discerned and mistaken for the whole truth. But even if the real core is penetrated and the organizing principle is discerned, the truth is too general for most teaching purposes. The result, therefore, is a one-idea presentation or too general a presentation, and is consequently either *fragmentary and false, or futile.*

But it is the purpose of this paper to point out an aspect of mathematical work which receives little or no consideration in any of the elementary texts. It is also the most important, by far, of all aspects of elementary mathematical study. It is, indeed, the phase of mathematical work for which all correct number teaching and arithmetic teaching prepare the pupil and the phase of mathematics whose non-existence would almost remove mathematical study from a rightful place in the school curriculum. I refer to the phase of mathematical study involved in *indirect measurement.*

Most mathematical teachers who make any attempt to square their practice with psychology base their teaching upon measurement. So far as the writer knows, no educator of any considerable experience claims that measurement is all there is to number study. As a matter of fact, direct measurement is very far from being all there is to number study. Those who read this claim into the statements of some of the leading writers on elementary mathematical teaching are in most cases open to the charge of wilful misrepresentation. Two things they need to learn: One is that a reputation for superior insight into the needs of the mathematical situation is not acquired through wholesale criticism of everything the "other fellow" does. The other is the more important reflection, abundantly borne out by the history of mathematical teaching, that whatever improvement comes to mathematical teaching originates from nonmathematical sources. The result is that mathematical teaching is about the only static element in all education. Almost everything else is subject to the requirement of adapting itself to the changed conditions of modern life. Mathematical teaching seems to be "the same yesterday, today, and forever." This is the one educational relic of the remote past which bids fair to remain with us for an indefinite future, so far as the mathematical priesthood is concerned. This perhaps accounts for, though it does not excuse, the notorious fact that mathematical teaching generally is the poorest of all the bad teaching of our schools. It is difficult to decide which works the more efficaciously to bring this deplorable result about—the elementary teacher's general ignorance of mathematics or the mathematical specialist's general ignorance of elementary school conditions. In the writer's opinion the damage wrought by the latter is the more positively harmful, because it always operates in conjunction with a conceit of knowledge which emboldens its possessor with the presumption of prescribing for conditions of which he is ignorant.

To anyone who is familiar with the facts it must seem clear that something is wrong with mathematical teaching; else there would be much less need of bribing students with mathematical scholarships and fellowships to prevent good men from leaving the study as soon as the subjects begin to become elective. Since a comparatively small number of students choose mathematics as a specialty after the first college year, it must also be clear that much of the antipathy of students to mathematics is engendered during their early school life. It is confidently believed that the number of voluntary elections of mathematical courses would be quadrupled in a very short time, without the use of rewards of any sort, by a general movement in the direction of improving mathematical teaching.

The laboratory method of instruction, which has already been used by some teachers of mathematical subjects for several years, promises much in the way of improvement. No one who has given this method a fair test ever abandons it of his own accord. Some have dropped it because certain superintendents, who would rather be in the majority than in the right, have forced its retirement in their schools. Too many distinctly good movements of recent educational advance have, however, been placed under ban by this official agency to make it a reflection upon the method.

It ought to be remarked, however, that a two-hour mathematical lecture inside of a physical laboratory is not what is meant by the laboratory method applied to mathematical teaching. Were it pertinent to the purpose of this article, it might be well to explain what is meant by the method applied to mathematics. But this is a matter of more concern to the secondary and university teacher than to the elementary teacher.

The aspect of mathematical work referred to above by the phrase "indirect measurement" is allied to laboratory work, and, as was said above, is altogether too much neglected in the elementary school. It has to do with that large body of problems in which the desired magnitude cannot be obtained by the direct application of the unit of measure to it. Such problems as the determination of the areas of plane figures and of the capacities of vessels and the volumes of solids are familiar examples of indirect measurement. In these problems we seek to ascertain how, by the direct measurement of lines and angles, we may infer the number of square or cubical units in the areas, or volumes. The direct measurement of a given area or volume would consist in selecting a superficial, or cubical, unit of magnitude *of the same sort* as the magnitude to be measured, applying it a definite number of times to the given magnitude and counting the number of applications. But this method is too inconvenient for practice and we substitute the indirect method of inferring the number of units from linear, or angular, measures which are more easily executed.

But while insisting upon the importance of more indirect measurement, in almost the same breath we must object to overworking the class of indirect measurements implied in the mensuration of the simple rectangle, and of the square-cornered box. Nowadays, we not infrequently witness the elaborate treatment of the rectangle in at least five of the grades of the elementary school. We seem never to get it taught, although two or three recitations would appear amply sufficient for all pre-geometrical purposes. In addition to this elaborate and repeated treatment we behold the rectangle doing all sorts of service in illustrating almost every new arithmetical operation from multiplication to the end. Nothing short of scarcity of number material or poverty of expedient on the teacher's part can excuse this overworking of a figure which is utterly simple and almost intuitively comprehensible. For the sake of the limitless field of problems exclusive of the rectangle, loosely called the *oblong*, let us give the rectangle a season of rest. After all it is the triangle, not the rectangle, which is the figure of fundamental mathematical importance.

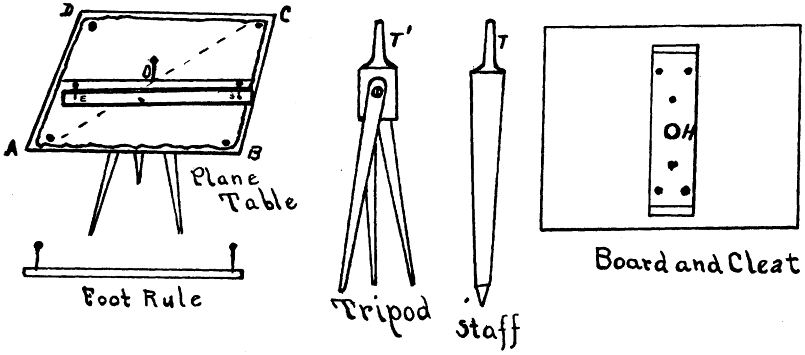

FIG. I

Elementary geometrical surveying furnishes another class of problems in indirect measurement of great interest and value to elementary pupils. The field work needed for this class of problems gives a vividness and a reality to this work which no amount or kind of pictured figure teaching can secure.

The only apparatus needed for this work are those shown in Fig. 1. They consist of a board 16 inches by 18 inches with a cleat on its back side, a tripod or staff, and a footrule with common pins stuck vertically near the ends. The cleat is provided with a conical hole to receive the tapering pins T or T'. The footrule with its pins for sights serves to direct the sight line.

To use the plane table, pin a sheet of paper on the board, stick a pin at some point, as *O*, and put the pin (*T′* or *T*) of the tripod, or staff, in the hole, *H*. Then set the plane table in the position shown on the left in Fig. 1, and it is ready for use.

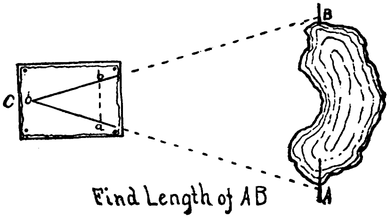

FIG. 2

With a carpenter's level, or a common phial partially filled with water, carefully level the board *ABCD*, and place the footrule upon it with its straight edge against the pin *O*, as is shown at *ES* in the figure. Sight past the pins at *E* and *S*, and turn the footrule around, keeping it against pin *O*, until the pins *E* and *S* are in line with some object, as a tree, steeple, or corner of a house. With a sharp lead pencil draw a line on the paper along the edge of the footrule. Now, without moving the board, turn the footrule around, keeping it touching the pin *O*, until the pin sights are in line with some other object. Draw a second line along the edge of the ruler. The angle at the pin *O*, on the board between the lines of sight to the two objects is now transferred to the paper. The magnitude of the angle may be obtained in degrees with the aid of a protractor.

Fig. 2 shows how a line, as *AB*, which cannot be directly measured may be measured indirectly. Set up the plane table over some point *O* on the ground from which the lines *OB* and *OA may be measured.* Pointing the footrule first toward *A* and then toward *B*, draw the lines *oa* and *ob* on the board. On any convenient scale, as 100 feet to the inch, mark off the length *oa* and *ob*. Connect *a* and *b* and measure *ab*. Remembering 1 inch represents 100 feet, find the length of *AB*.

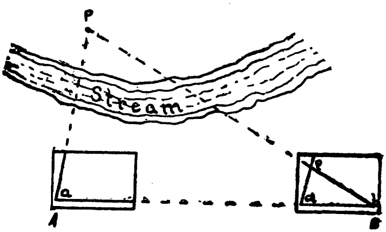

FIG. 3

Fig. 3 shows how the plane table may be used to find the distance from two points *A* and *B* to an inaccessible point *P*.

Set the board up over point *A* on the ground, level the board, and with the footrule draw lines from a point *a* on the board between *P* and toward a stake stuck at any other *accessible* point, as *B*. Move the tripod to a position over *B*, place the straight edge of the footrule on the line *ab* and turn the board around *on the pin T′* until the sight pins are in line with a stake, now stuck at *A*. Using any convenient scale, mark off *ab* long enough to represent *AB*. Stick a pin at *b*. Place the footrule against this pin and sight it into line with *P*. Draw *bp*. Measure *ap* and *bp* and, remembering the scale, the distances *AP* and *BP* at once become known.

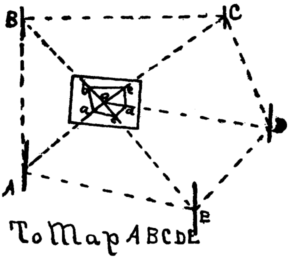

FIG. 4

Fig. 4 shows how the plane table, set up at a single point within an area *ABCDE*, enables a student to map, or plat, it by laying off to scale the distances *oa*, *ob*, etc., to represent the measured distances *OA*, *OB*, etc. It is only necessary to connect the points *abc*, etc. All the sides of the polygon *ABCDE* become known. If the area is wanted it is only necessary to measure the bases and altitude of the little triangles on the board and apply the ordinary rule for the area of a triangle. If the paper used is squared paper the area may be found still more simply. The method will readily suggest itself to the reader.

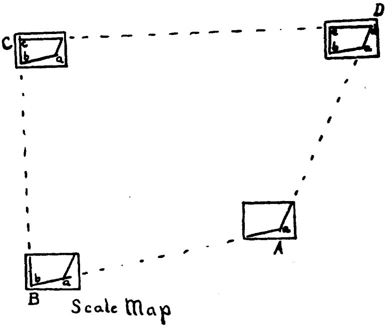

FIG. 5

Fig. 5 shows how the outline of any irregular tract may be found by setting up the board at its corners. Let the corners of the tract to be platted be *ABCD*. Set the board up over *A*, level it, and from some point, as *a*, on the board direct a line with the sight-rule toward a stake at *D* and then toward another at *B*. Then move the board forward and, placing it over *B*, level the board. Set the sight-rule along the line *ab* and turn the board around pin T' until the sight-pins are in line with a stake at A. This brings the board parallel to its position over A. This step is called *orienting* the board. Lay off ab to represent AB to scale. Place the sight-rule on a pin stuck at b and line it in with a stake at C. Draw a line on the board along the rule from b toward C. Now move the board to C, level it, orient it, lay off bc to represent BC to the same scale and draw a line from c toward D. This gives on the board a small polygon $abcd$ similar to $ABCD$.

Engineers call this process *traversing* with the plane table. If squared paper is used the area is easily found by counting the squares. If not, resolve the figure into triangles and find the sum of the areas of the triangles.

Numerous other uses of this board will readily suggest themselves from what has been said here.

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